PHY 3: Lab Manual

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This lab manual is being used at C.W. Post, part of Long Island University. It is made available online, and is freely usable. It is written in LaTeX, and the source is available upon request. If the materials here are extended or corrected, we ask that such additions and changes be sent back to the authors so that everyone can benefit from the collective efforts of all users of this document. It originated at Southampton College.

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Preface

Lab is a special time, requiring from you something quite different than most lecture classes. You have to participate, think, and fix. Many students are frustrated by this, seeing any problem as a conspiracy to delay their departure. First instinct is often to blame the equipment. I've taught at a variety of campuses, with old and new equipment, and the refrain is always the same, "why can't you get working equipment?" Equipment, good or bad, does not always work the way we expect. That does not necessarily mean the equipment failed. Maybe you're not doing something properly, or maybe the Universe simply isn't cooperating.

My favorite analogy for teaching physics is basketball. The coach tells you how to aim, how to execute a proper shot, but you miss. Did the ball have too much air and so it bounced off the rim? Is the backboard too tight? A player doesn't make excuses, and, for the most part, you shouldn't in lab either.

This guide is meant to provide you with information to execute the various labs properly. It doesn't spell out every detail. You'll need to fill in the gaps. Suggestions are welcome, but you need to realize that you're expected to think in lab. Remember, I have no need of 25 lab reports. You're doing the lab for the experience of doing it.

What are you supposed to get out of the lab? You are supposed to see whether the Universe behaves the way we (the book and I) are telling you. Is energy conserved? Do oscillating systems have a natural, resonant frequency?

Can you do the labs without observing the behavior of the Universe? Certainly. So many of the lab writeups I see have all the data and calculations done correctly, but from the written discussion, it is quite clear that the student took no time to think about what it all means. Such failures aren't a matter of intelligence, and it perhaps isn't laziness. So many students have been drilled incessantly for so long, that the classroom is the last place they would think about actually using their brains. But please try.

What else should you get from lab? Well, there's the little matter of seeing what science is all about. You are supposed to be learning about what it takes to collect data, analyze it critically, draw conclusions from this data, and defend it against criticism. Science is this process. You should be doing it in biology, in chemistry, and in physics.

Remember, coming to lab and executing the instructions are only the beginning. Junior high kids can do that. The hard stuff happens once you've followed instructions and collected the data.

Contents

Chapter 1

Measurement

1.1 Introduction

As the first lab for this class, we stress here the skills needed for future labs, namely taking and recording measurements and scientific writing. For this reason and because we haven't yet covered much material, this lab is relatively straightforward, yet fundamental. Furthermore, this lab will serve as an introduction to error analysis.

We'll be measuring the density of various objects, a property of the material not its shape. We can then compare various objects to determine whether or not the objects are made of the same material. Such a judgment of the material requires an argument to be made because it is highly unlikely that two different measurements will yield exactly the same value.

Hence, in your writeup your mission is to present data effectively to support your argument as to whether any of the objects are made from the same material.

1.2 Procedure

1.2.1 Measure the geometry of various objects

For each object, measure and record with appropriate tools the dimensions so that you can then calculate the volume. Additionally, estimate the uncertainty ("error") for each measurement; for a ruler, this might be .5 mm.

Keep in mind that the volumes of various shapes are:

$$
\begin{aligned}
V &= l \times w \times h \quad \text{right parallel piped (3D rectangle)} \\
V &= \frac{4}{3}\pi r^3 \quad \text{sphere} \\
V &= \pi r^2 \times h \quad \text{right cylinder} \\
V &= \frac{1}{3}\pi r^2 \times h \quad \text{cone}
\end{aligned}
$$

Also remember, in your writeup present all original data and uncertainties along with computed quantities, preferably in a table. Don't forget to record the uncertainties in these measurements (i.e. 5 cm ± .05 mm).

Make sure that, when possible, you use the ruler, the vernier caliper, **and** the micrometer for the different measurements—they have different levels of precision. The idea is to have data from all the devices so that you can compare them.

1.2.2 Measure the mass of the objects

Using the triple beam balance as well as the electronic balance, measure and record the mass of the various objects (remember to record the uncertainty as well).

1.2.3 Record the data

After you've recorded the data, think about how best to present it to the reader. Can you put the measurements of geometry of different types of objects together? Can you also include the mass measurements?

At this point, you also need to calculate the (mass) density of the various objects by dividing their mass by their volume. Pay attention to units. Make sure that the units of density for all the objects are the same.

1.3 Writeup

Keep in mind that the goal here is for the lab writeup to be similar (but less work!) to a scientific paper. Thus, do not say that "the point of the lab was to learn about...".

Start the lab with the **point of the lab**. In paper, you'd generally introduce the topic just a bit, but that's not necessary here. Examples for Physics 202 labs include:

1. Using an electron gun, we measure the charge to mass ratio for an electron.

2. By plotting equipotential lines determined with a galvanometer, we investigate the geometry of electric field lines around various conductors.

3. We construct a low-pass filter using resistors and capacitors.

In paper, this clearly serves to tell the reader why you're doing the lab, but it also serves to tell me whether you understand the importance of the lab.

Follow this with the **data**. Many people underestimate the importance of presenting this clearly with forethought. You want to put as much into tables as possible to make it easy to read. In many labs in high school you might have gotten worksheets with tables which you just fill in. However, you need to develop the skill to determine how to organize the tables yourself.

Perhaps the hardest part is to **analyze the data** and draw conclusions from it. Usually one starts with simple observations about the paper. Don't assume that the reader is going to see what you say. For example, if you say some set of values are all the same, will the reader agree? Instead say something more detailed such as "the difference between the minimum and the maximum values was only 0.01 cm which is small compared to...."

I usually provide you with questions for each lab. These aren't to be answered like a worksheet. Instead they are to guide you in terms of what you want to analyze.

1.4 Guiding Questions

1. Which objects do you suspect are made from the same material?

2. Can you determine certain objects to be made of different materials?

3. Are the rulers more inaccurate than the calipers? Are they less precise?

4. Is the balance more inaccurate than the electronic scale? Are they less precise? Do the measurements from the various devices agree with each other?

5. Look up the densities in your text or elsewhere. How confident can you be that you've identified the correct material for each?

Chapter 2

Static Forces

2.1 Introduction

So far we've studied a few things which we'll need later, but we have yet to get to some "meat" of physics. We covered units which we'll obviously need in our studies. We covered kinematics, the equations describing motion. And we've introduced vectors. However we really haven't gotten to anything that can be considered something other than math. Math is the language of physics, and we have to speak it in order to do physics. But, math does not ask "why": why does something travel in a line? why do vectors work so well at describing so much?, etc.

In this lab, and quite soon in lecture, we introduce the first of our physics. We don't start small though. In fact, we start with arguably the most important physics ever, namely Newton's Laws. Mastery of these laws can get you to the moon, a job as an acrobat, promotion in the army, and plenty else.

Newton has three laws; we only need one for now. The relevant law says that if you add up all the forces on an object (the *net force*) and get zero for the sum, then that object will have zero acceleration. So, if no force, then no acceleration. If I pull on a rope really hard and you pull in the opposite direction, then the two forces cancel out. So the net force is zero and Newton says the rope doesn't accelerate. Does this fit your intuition for the situation described?

Here, we'll be testing that law. One complication is that force has a magnitude (*how hard do you pull the rope?*) and a direction (*in which direction* *do you pull?*). Thus, we represent forces as vectors. This makes writing down Newton's Law as an equation pretty easy:

$$\Sigma_i \vec{F}_i = 0$$

The symbol Σ (spelled "sigma") simply means to sum a bunch of numbers where i simply counts the things you're adding (1, 2, ...).

To test it, we'll apply forces to one particular object until we make the acceleration of that object zero. Then Newton tells us that the vector sum of these forces should be zero. To sum the vectors, we have to measure their magnitudes and directions.

The equipment we'll use is called a *force table*. In the center of the platform is a small ring which is the object we want not to accelerate. To apply forces, we'll attach strings to the ring and hang masses on these strings. The masses then hang over the pulleys. There will be one force per string and the magnitude of that force will be computed by multiplying the weight of the mass hanging on the string by g. The direction can be measured from the printed angles on the platform itself.

2.2 Procedure

2.2.1 Lab Execution

1. Make sure the platform is level.

2. Setup the platform with the ring in the center held in place by the pin. Don't remove the pin until you've balanced the ring so that when you remove the pin, it stays still.

3. To balance the ring, first attach mass hangers to the various strings (one for each pulley present). Before doing so, confirm the mass of the hangers themselves by measuring their mass.

4. Make sure that the strings point toward the center of the ring.

5. Secure the pulleys at various angles. **Do not** place the pulleys directly opposite each other. Measure and record the angles. Be sure to note where the angle 0° is, and in what direction the angle increases (CW or CCW). Also note the uncertainty σ_θ in measuring each angle.

6. Add masses to the hangers to balance the ring. One can also adjust the angles, but then you'll have to measure them again. It may be best to measure angles after things are balanced.

7. Be sure to perturb the ring frequently when determining whether the setup is balanced. Otherwise, the pulleys tend to stick and mislead you into thinking things are balanced.

8. Record which string has how much mass attached to it. Include the uncertainty σ_m in the mass measurement.

9. Repeat for another set of angles and with more mass on the strings.

10. While your setup is balanced, add a 5 g mass to one of the strings. Is it still balanced? How much mass can be added with the setup still being balanced? What does this exercise tell you?

2.2.2 Analysis

1. Sketch the vectors with appropriate axes.

2. Using tables, compute the magnitudes and $x-$ and $y-$ components of the various vectors.

3. Compute the components of the net force on the ring $\Sigma_i \vec{F}_i$.

4. Compute the magnitude of the net force. Also, estimate the uncertainty in this magnitude (show your calculation).

2.3 Questions

1. Does it matter that the platform is level?

2. Pulleys are not perfectly free to rotate; there will always be some resistance. How would this imperfection impact your results?

3. Chances are that your net force won't be exactly zero. How close do you expect it to be? Do you think your results provide supporting evidence for Newton's law, or do they refute the law?

4. To what extent might the string cause error? If it stretches, does this cause error?
5. Would you expect the error to be greater or less if you increased all the masses on the strings and rebalanced?

Chapter 3

Mechanical Advantage

3.1 Introduction

Pulleys are an interesting type of machine. They're simple enough for easy investigation and yet at the same time they demonstrate some important concepts of physics. They also serve as a transition from describing systems with forces and Newton's Laws to describing systems with *work* and *energy*, a transition we'll be making in lecture within the next couple weeks.

As an introduction, consider the simplest pulley system, in particular the $n = 1$ system where n describes the number of pulleys being used (see Fig. 3.1. If on one end of string we place some mass M (the *load* to be lifted), then we can pull on the end of the pulley (after draping it over the single pulley) to lift the load. The load will have weight $F_w = mg$. We can measure the lifting force F_l using a spring scale. If the pulley is ideal (no friction or resistance to turning) and the string doesn't slip, then you should be able to reason that $F_l = F_w$.

Does that mean the pulley doesn't help us? After all, we still have to lift with the same force as if we just grabbed the load in the first place.

We next consider the $n = 2$ pulley system. You will be experimenting with various n-pulley systems to measure what they do, but you need also to be able to analyze and predict their behavior.

In addition to computing the weight F_w lifted and the lifting force F_l, you will also be computing two other quantities. The first is *mechanical advantage*, MA, defined by

$$MA \equiv \frac{F_w}{F_l}.$$

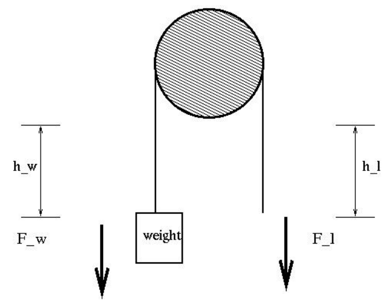

Figure 3.1: Diagram of an $n = 1$ pulley system.

The second is *work* done, W, defined by:

$$W \equiv F\ h$$

where h is the distance over which the force acts. Thus we can define the work done in lifting as $W_l = F_l\ h_l$ and the work of the weight as $W_w = F_w\ h_w$.

In our normal SI units, work has units of $N \cdot m$ which has a special name, *Joules*. This is a unit of energy, and indeed work is a form of *energy*. You might suspect energy is conserved from either past experience or common sense, and so you'll investigate if such is demonstrated here.

You should get the following out of this lab:

1. Understand *how much* the pulleys benefit in lifting.
2. Understand *why* the pulleys benefit in lifting and be able to analyze it.
3. See how energy comes into play here, and see the beauty of how the analysis yields its conservation.

3.2 Procedure

3.2.1 $n = 1$ Pulley System

As we do in most of these labs, we start by looking at the simplest system. We can usually get a good handle on what *should* happen here, so the simplest system is usually one we can get an idea of what kind of error to expect in the more complicated situations.

1. Set up the $n = 1$ system (as in Fig. 3.1) with some weight whose mass you measure. Record the uncertainty in the mass.
2. Using the spring balance, record the minimum force necessary to lift it and the uncertainty.
3. Record the distances you pull the string and the distance that the object moves. Record the uncertainties.
4. Compute the MA and the uncertainty; compare to what you expect.
5. Repeat for 3 different masses.

3.2.2 $n = 2$ Pulley System

1. Set up the $n = 2$ system and repeat the procedure from the $n = 1$ system.

3.2.3 $n = 3$ Pulley System

1. Decide how to setup this system and sketch it in your lab report.
2. Set up the $n = 3$ system and repeat the procedure from the $n = 1$ system.

3.2.4 $n = 4$ Pulley System

1. You do **not** need to set this system up.
2. **Predict** what the mechanical advantage for this system will be.

Sample table

$m_w(kg\pm)$	$F_l(N\pm)$	$h_l(m\pm)$	$h_w(m\pm)$	$F_w(N)$	MA	$W_w(J)$	$W_l(J)$	%e (MA)	% (W)

Chapter 4

Velocity and Acceleration: Air Track 1

4.1 Introduction

The motion we study is a somewhat idealized picture. In the real world, objects rarely have constant acceleration because of friction, air resistance, or any of many other factors. However, we need to start somewhere, and many times we can ignore friction.

To explore such kinematics we will work with *gliders* on an *air-track*. The air-track is basically a linear air-table similar to an air hockey setup. Air blows through tiny holes and these provide a cushion of air on which the glider moves.

Obviously they're not perfect at ridding experiments of friction. But they do well. In addition to studying kinematics in this lab, we want to get a good understanding of how much friction remains because we will use these air tracks later when we study collisions.

The air tracks are pretty simple. Plug in the air blower to the end of the track, air comes out the holes, and you put the glider on. There are, however, a few things to mention:

1. Don't put the glider on the track unless the air is on, otherwise it'll scratch both surfaces.

2. Consider your release of the glider. Don't push down on it, or its release won't be smooth.

3. Be aware whether you're supposed to release the cart with no velocity or not.

4. Consider whether the track is level. The track can be "not level" in two ways. Many times we'll have it inclined on purpose, but otherwise you'll want to check that it is level. Also consider whether the track is leaning to one side. If so, this will increase friction as well as being generally unstable.

Perhaps the more difficult task is to master the *photogate timers*. To actually study the motion, we'll need to be able to measure times and speeds. That's what the timers are for. They work by having light sent across a space (the *gate*) and are triggered when that light beam is broken. So it can time how long a glider takes to cross the gate or it can time how long to go from gate to another.

There are two modes that concern us and the mode is set with a switch on the timer unit:

1. GATE mode – the timer records for how long the beam is blocked. You can compute a velocity by taking the length of the object blocking the beam and dividing by the timer reading.

2. PULSE mode – the timer starts timing when the beam is first broken until it is broken a second time. You can compute a velocity by taking the distance between the two gates and dividing by the timer reading.

Keep in mind, that two gates can be connected to a timer unit. So in gate mode, you can block either gate. In pulse mode, you can start timing at one gate and have it stop by blocking the second gate. Also remember that if you're measuring velocity with these gates, *do not continue to push a glider* while it has entered the light beam of a gate. This will mess up your reading because your hand may be accelerating the object while you're trying to measure the velocity.

Another feature of the timers is that they have memory. We will be using this much of the time and it resembles "lap time" on a stop watch. For example, say we're in GATE mode. A glider goes through the gate and the timer shows the time it took the glider to get through the gate. But now a second glider goes through the gate. The timer reading doesn't change but the timer did measure this event. If you hit the toggle switch you read what's stored in memory. This time is actually the time for the second glider added onto the time for the first glider. So, to get the time for the second one, you must subtract the first reading shown. This lets us use the timers to measure two readings which is often important.

Here are some important points about the timers:

1. In general, release a glider before it enters a photogate.
2. Make sure the gate cables are pushed *completely* into the appropriate socket otherwise they won't work.
3. Check that the times you get are reasonable. Often, if the height of the gate is too high or otherwise incorrect, all measurements will be about $0.014s$ which is *totally wrong*. What we measure does not move that fast!. This is an example where you need to pay attention to your data while you take it so that you can correct a problem.

4.2 Procedure

4.2.1 Deceleration on a level track

1. Level the track!
2. Setup two photogates a distance D apart (about $50 - 70cm$) on the track with the timer set to GATE mode. You'll have to use the memory feature. Measure the distance D between the photogates and its associated uncertainty.
3. Send a cart (and measure its length L_1 and uncertainty) through both gates at a reasonable velocity.
4. Measure, record, and compute the velocity of the glider and the uncertainty in the velocity at both the first (t_1, v_1) and second (t_2, v_2) photogate timer. You'll need to state the uncertainty in the times as well.
5. Repeat four times (twice in each direction).
6. Compute a percent difference for each case. Make sure to retain the sign of the percent difference. In other words, the percent difference should reflect whether the cart sped up or slowed down.

Sample table

$D =$ $\pm$	$L_1 =$ $\pm$				
Direction (L or R)	t_1 (s) $\pm$	t_2 (s) $\pm$	v_1 (cm/s) $\pm$	v_2 (cm/s) $\pm$	% diff.

4.2.2 Velocity versus distance

1. Incline the track at about 2–10°.
2. Setup a single photogate near the low end of the track set to *gate* mode.
3. Place the glider a distance $D = 5$ cm along the track above the photogate, and release.
4. Measure, record, and calculate the timer reading and velocity of the glider (be careful here to do it as in part 1).
5. Repeat the measurement a few times to get a good average.
6. Repeat from step (3) increasing the distance D from the gate increasing by 5 cm (so $D = 5$ cm, 10 cm, 15 cm, ...) all the way up the air track.
7. Make a plot of the average velocity, v_{av}, versus D.
8. Try plotting some other function of v (say $\sqrt{v}$, $1/v$, v^2, etc.) versus D until you arrive at something that looks linear.

Sample table:

estimated angle = $\pm$					
D (cm) $\pm$	t_1 (s) $\pm$	t_2 (s) $\pm$	t_3 (s) $\pm$	t_{av} (s)	v_{av} (cm/s)

4.2.3 Determining g

1. Incline the track at about 2–10°.

2. Measure the height and length of the incline and compute the angle θ either via

$$\theta = \tan^{-1}\left(\frac{\text{height}}{\text{length on table}}\right)$$

   or

$$\theta = \sin^{-1}\left(\frac{\text{height}}{\text{hypotenuse}}\right)$$

   Make sure that the length and height correspond to sides of a right triangle formed by the track and the table. In particular, measure the height of the track as compared to the position at which the track is level. Estimate the uncertainty in the angle.

3. Set up two gates some distance away and measure the distance, D, between them. Put the timer in GATE mode.

4. Release the glider above both the gates, and record the timer measurements. Compute the velocity v_1 at the top gate and then the velocity at the lower gate v_2.

5. Repeat for different gliders (record lengths L) or with extra mass on the glider (record the entire mass of glider and masses used, m).

6. For each case, compute the acceleration via

$$a = \frac{v_2^2 - v_1^2}{2D}$$

7. If the track were vertical, then we would have $a = g$. If the track were perfectly level, then $a = 0$. It turns out that for a given inclination angle, $a = g\sin\theta$. For each case, compute g via:

$$g = \frac{a}{\sin\theta}$$

8. Compute an average value of g and the corresponding percent error.

Sample table:

height =	$\pm$	length =	$\pm$
$\theta =$	$\pm$		

L (cm) $\pm$	m (kg) $\pm$	D (cm) $\pm$	t_1 (s) $\pm$	t_2 (s) $\pm$

...

v_1 (cm/s)	v_2 (cm/s)	a (cm/s^2)	g (cm/s^2)

4.3 Questions to think about

These do not have to be answered and turned in. However, I do want you to read these questions before you execute the lab so that you can think about important things and get the most out of the experience. You can also use these questions to review for the final.

Section 4.2.1:

1. In the ideal case (no friction, air resistance, etc.), what would be the relationship between v_1 and v_2?

2. What does the percent difference mean? How would you make use of this information in other labs/measurements with the air track?

3. Is it a large percent difference? Would it be bigger or small if we used gliders on wheels? Gliders on a wet surface? How good a job is the air track doing in getting rid of friction?

4. Is v_1 greater than or less than v_2? Is this expected? If it's not expected, can you explain it?

5. Why should you do this in both directions? Would you expect a different result? Did you see a different result?

6. From these results, can you estimate roughly the expected effect of friction for other experiments executed on these air tracks?

Section 4.2.2:

1. Why was the plot of v versus D *not* linear? Why *was* the plot of what you chose linear?

2. Do you see an effect due to friction? Can you determine approximately how big an effect it would be? If you don't see an effect, what effect would be apparent if friction played a larger role?

Section 4.2.3:

1. What angle would be the best to use? Can the angle be too big or too small?

2. In which quantity are you most uncertain?

3. Was the overall uncertainty in g large or small?

4. Was this experiment a success? Do you accept the "accepted" value of $9.8\ \mathrm{m/s^2}$?

5. How would friction affect your results? Did it affect your results?

Chapter 5

Hooke's Law

5.1 Introduction

Continuing with forces, we now encounter the force of the spring. The force due to a spring is called a *restoring force* because the force it exerts is that which opposes a deformation. Hence, if you compress the spring, the spring pushes to elongate itself. If you elongate the spring, it pulls back to compress itself to its natural length. So, if the force exerted by the spring is F and deform it some distance x, then the fact that it is a restoring force implies that F and x will have opposite signs.

Another property of the spring is that for small displacements x, the force is linearly related to the deformation. In other words, $F \propto -x$. This property shouldn't be taken for granted, and in this lab we'll explicitly deal with a rubber band which is not expected to have this property (but which should nevertheless exert a restoring force). We can go a step further, and write down Hooke's Law for the spring

$$F = -kx$$

where we have put in a constant of proportionality so that we can write the relationship as an equation. This constant reflects the intrinsic properties of the spring and is called the *spring constant*. Roughly, it's the *stiffness* of the spring, but it doesn't depend on x.

In the first two procedures below, you should be certain to note:

1. whether the rubber band has a force linearly related to the displacement

2. whether the spring has a force linearly related to the displacement.

You should also recall in the last lab dealing with pendulums, we encountered simple harmonic motion. I wrote there that harmonic motion was everywhere, and indeed we encounter in the spring. If one puts a mass m on an upright spring, deforms it, and then releases it, the mass executes simple harmonic motion moving up and down. The theoretically predicted period follows the relationship

$$T = 2\pi\sqrt{\frac{m}{k}}.$$

Thus, if we measure the period for various values of m, we should be able to obtain another measurement of k and compare to the value obtained earlier. Regardless of the value however, one should note from their data to what extent this relationship holds.

5.2 Procedure

5.2.1 Rubber-band

1. Suspend a rubber-band on the spring apparatus (carefully remove the suspended spring first).

2. Attach a mass hanger with appropriate mass and note the displacement (look at how far the bottom of the mass hanger moves when you allow the band to stretch).

3. Record the displacement y_i and the total mass m_i (including the mass of the hanger).

4. Repeat for at least 6 different masses which span an appropriate range.

5.2.2 Spring

1. Suspend the spring on the spring apparatus.

2. Attach a mass hanger with appropriate mass and note the displacement.

3. Record the displacement y_i and the total mass m_i (including the mass of the hanger). Also, compute the force of the spring $F_s = m_i g$.

4. Repeat for at least 6 different masses which span an appropriate range.

ANALYSIS: Plot the force of the spring $F_s = mg$ (vertical axis) versus the displacement y (horizontal axis) for both the rubber band and the spring. The plot can be by hand or on the computer. Is the plot for the band linear? What about for the spring? Add a best fit line to the spring points. From the slope of this graph ($F_s = ky$), obtain a value for the spring constant k in units of N/m.

5.2.3 SHM

1. Suspend the spring on the spring apparatus.
2. Attach a mass hanger with appropriate mass. You don't want too much mass nor too little where the masses bounce into the air.
3. Stretch the spring and release.
4. Time an appropriate number of cycles from which you can obtain the period from division.
5. Repeat for at least 6 different masses.
6. Plot T^2 versus m. Add a best fit line. Obtain a second value of k. Compare it to your earlier obtained value of k.

RUBBER-BAND:

i	$m_i(g \pm \quad)$	$y_i(cm \pm \quad)$	$F_s(N)$
1			
2			
3			
4			
5			
6			

SPRING:

i	$m_i (g \pm \quad)$	$y_i (cm \pm \quad)$	$F_s (N)$
1			
2			
3			
4			
5			
6			

SHM:

i	$m_i \ (g \pm \quad)$	# Oscillations	$Time \ (s \pm \quad)$	T (s)
1				
2				
3				
4				
5				
6				
7				
8				

Chapter 6

Centripetal Force

6.1 Introduction

In circular motion, one has acceleration. It is this acceleration which continually **turns** the object from what would otherwise be a straight line path. To provide this acceleration, one needs a force, called the *centripetal force*.

In this lab, we want to study the case of circular motion. In lecture, we can determine how much force is needed to keep an object of a certain mass M moving in a circular orbit with radius R with period T. But is this correct?

In this lab, we'll measure the parameters above (M, R, and T) and therefore we'll be able to compute the theoretical amount of centripetal force $F_{\text{theoretical}}$ needed. However, we'll also measure directly the force being used as the centripetal force $F_{\text{experimental}}$. By comparing these two values, we'll be able to conduct a test, but what are we actually testing?

As a side note, I should mention that circular motion often confuses students who picture circular motion as some sort of "steady" motion because things seem to stay the same. Some students therefore assume that forces balance out. They don't! Remember we have continuous acceleration which means the forces do not balance. The other confusion is that students often think that the centripetal force exists in addition to the other forces in the setup. That's not true. Think of "centripetal force" as a title, like "President of the United States." If we have President Bush in a room, there's only one person there, not the President **and** George!

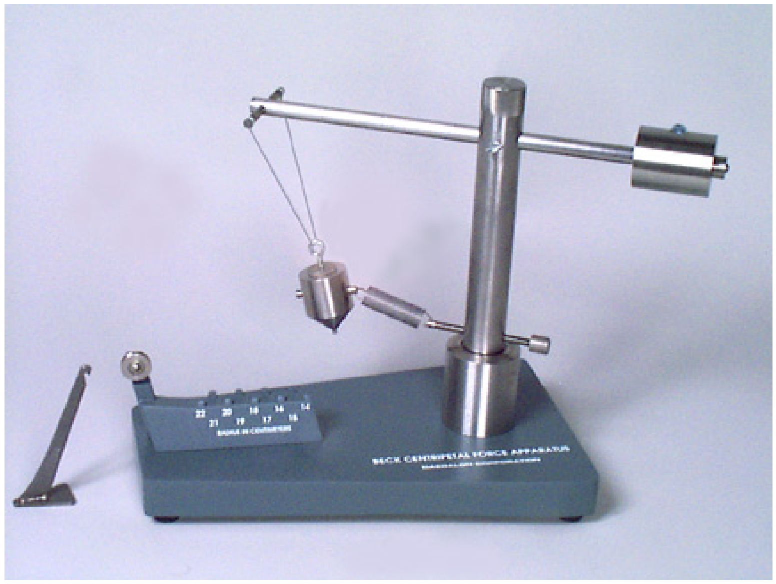

Figure 6.1: Picture of the centripetal force apparatus.

6.2 Procedure

1. Measure the mass of the bob along with your uncertainty in its measurement.

2. Fix the pointer at a certain position by tightening the screws on the baseplate. This fixes the radius R of the circular path of the bob so measure this radius as the perpendicular distance from the pointer to the axis around which the bob spins.

3. Hang the string that is attached to the bob over the pulley. Add enough mass to the hanging string so that the spring stretches enough that the bob sits directly above the pointer. Make sure the string is horizontal and the bob suspension is vertical.

4. Make sure that the string on which the bob hangs is perpendicular to the rotating bar. To do this, you will have to adjust the length of the bar at the top of the apparatus.

5. Measure the amount of hanging mass m, and compute the tension in the string, $F_{\text{experimental}} = mg$. This tension is the experimental value

because it's a measurement of the force needed for the circular path.

6. Remove the string and hanging mass. Manually spin up the bob so that it traces a circular path which carries the bob directly over the pointer. You'll need to keep spinning the axis to counter the effects of friction, but try to reach a steady state.

7. Then measure the period. The best way to do so is to choose some number of revolutions N and measure the time t it takes for the bob to complete these. Then divide to get the period $T = t/N$. Take a few of these measurements so that

   - You get a good average to use in your measurements.
   - You get a good idea of your uncertainty from the spread in values.

8. Compute the speed of the bob v. Since the bob traverses the circumference of a circle for each period, the speed is then $v = 2\pi R/T$.

9. Compute the centripetal force $F_{\text{theoretical}} = Mv^2/R$ where M is the mass of the bob.

10. Compute the percent difference between the experimental and theoretical values of the centripetal force.

11. Repeat for a different position of the pointer.

Chapter 7

Ballistic Pendulum

7.1 Introduction

The ballistic pendulum provides a method to measure the velocity of a projectile. We shoot a projectile horizontally so that initially we're only dealing with linear momentum. However, we arrange to have this projectile lodge itself in a pendulum so that the projectile's momentum gets transferred to the pendulum/projectile system. That's fine we can track that using the conservation of linear momentum.

The pendulum/projectile system then moves upwards. This converts kinetic energy into potential energy. If we can measure this potential energy, then we know how much kinetic energy we had.

The end result is that using *both* momentum conservation and energy conservation, we arrive at an equation relating the height to which the pendulum reaches with the initial velocity of the projectile.

Let's define some terms before we get too far into the equations. We'll call the projectile's mass m, in contrast to the mass of the pendulum bob, M. Likewise, the projectile's initial velocity we'll call v. Then pendulum ends up with no velocity, but we'll call it's velocity just after the projectile embeds itself in it V. Finally, the height to which the pendulum climbs we'll call h.

Thinking only of the collision, the conservation of linear momentum tells us that the momentum of the projectile (mv) goes into that of the pendulum/projectile system ($(m + M)V$):

$$mv = (m + M)\,V. \qquad \text{(momentum conservation)} \tag{7.1}$$

Figure 7.1: Picture of the ballistic pendulum apparatus.

Conservation of energy equates the kinetic energy of the pendulum/projectile system ($1/2(m+M)V^2$) with the potential energy later ($(m+M)gh$):

$$\frac{1}{2}(m+M)V^2 = (m+M)gh. \quad \text{(energy conservation)} \tag{7.2}$$

If we solve the first equation for V and plug it into the second, we arrive at the sought after relationship

$$v_I = \frac{m+M}{m}\sqrt{2gh}. \quad \text{(method one)} \tag{7.3}$$

This whole process leaves us with a measurement of the initial velocity of the projectile, but we have not tested anything yet. We want to test to see how accurate this measurement is. Perhaps energy is not conserved. Another way we can get the velocity is to shoot the projectile horizontally with *no* pendulum. The projectile will shoot off and hit the floor. If we can measure the distance the ball travels horizontally X along with the distance it falls vertically Y, then our knowledge of projectile motion can tell us the initial velocity.

We look first in the vertical direction. The projectile has zero initial vertical speed and falls distance Y in a time t with acceleration g:

$$Y = \frac{1}{2}gt^2. \tag{7.4}$$

In the horizontal direction, the projectile has constant speed (why constant?) and goes a distance X in a time t (the same time as above):

$$X = vt. \tag{7.5}$$

If one combines these two equations, you can then solve for v:

$$v_{II} = \sqrt{\frac{gX^2}{2Y}}. \quad \text{(method two)} \tag{7.6}$$

7.2 Procedure

7.2.1 Part I:

1. Measure the mass of the projectile, m.
2. Measure the mass of the pendulum, M.
3. Fire the projectile 5 times, and measure the vertical distance it travels upward, h.
4. Calculate the value of initial v_I.

7.2.2 Part II:

1. Move the pendulum out of the way of the projectile.
2. Fire the projectile horizontally onto the floor. Use carbon paper over white paper on the floor to mark how the projectile traveled.
3. Measure the horizontal distance, X the projectile traveled.
4. Measure the vertical distance, Y the projectile fell.
5. Determine the initial horizontal velocity, v_{II}.
6. Compute a percent difference between v_I and v_{II}.

Chapter 8

Moment of Inertia

8.1 Introduction

The moment of inertia of an object is a measure of how difficult it is to get an object spinning (angular acceleration) in much the same way that mass is a measure of how difficult it is to accelerate (linear acceleration) an object. One difference though, is that we assume mass to be fundamental whereas we can calculate the moment of inertia from how the mass of an object is distributed. When a skater pulls her arms inward during a spin, her mass stays the same but its distribution changes, as does her moment of inertia.

In this lab we'll measure the moments of inertia for a few different objects by spinning them up and measuring the appropriate dynamical quantities. We can compare these experimental results to calculations of the moments from standard formulas which only need to know the geometry of the objects.

The dynamical measurement follows from an argument we present here. We use a **driving mass** m which we hang from a cord a distance h from the floor. The driving mass takes a time t to fall, during which its average velocity obeys

$$\bar{v} = \frac{h}{t}. \tag{8.1}$$

The average velocity is just half of the final speed v because it starts from rest. Hence, we have the relationship

$$v = \frac{2h}{t}. \tag{8.2}$$

The cord spins the **drum**. If we denote the radius of the drum as R, then the angular velocity ω follows

$$\omega = \frac{v}{R} = \frac{2h}{Rt}. \tag{8.3}$$

With these relations, we now consider that energy should be conserved for this system. Initially nothing is moving and the energy is all potential. In the final situation, the driving mass is just about to hit the floor and the drum is spinning. Hence, the final energy is all kinetic and we have

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2. \tag{8.4}$$

Here, I denotes the moment of inertia of the drum and everything attached to it. With some algebra and the various relations above, the moment of inertia becomes

$$I = mR^2 \left[\left(\frac{gt^2}{2h} \right) - 1 \right]. \tag{8.5}$$

In this lab, the idea is then to time a falling mass which spins the drum and anything mounted to it. In the first trial, nothing will be attached, but you will repeat with different objects on top. For each of these, you'll use the equation above to compute the moment of inertia.

8.2 Procedure

1. Measure the diameter of the drum and compute its radius.
2. Execute/repeat the following for each case of: (i) cross, (ii) cross plus ring, (iii) cross plus disk (iv) ring plus disk.
3. Determine (and record) the *compensating mass*, the mass needed to counterbalance the frictional forces. The compensating mass provides just enough force to keep the drum rotating at constant angular velocity and should be in the range from 0 to 30 grams.
4. Determine an appropriate amount of driving mass for each case. Too much mass and the system will move too fast for good results. Too little will result in friction playing a large roll and ruining your results. Keep in mind that the driving mass should be recorded as the total amount of mass on the string minus the value you have determined as the compensating mass.

5. Measure the distance the driving mass falls, h. Do **not** let the mass reach the floor because then the string will get tangled. Instead have someone stop it at the distance you measure.

6. Measure the time t for the driving mass to fall the distance h. Take and record five measurements, using the average in your calculations.

7. Compute the moment of inertia using the dynamical equation presented above for each case.

8. Compute the moments of inertia of just the ring using

$$I_{\text{ring}} = I_{\text{cross+ring}} - I_{\text{cross}}$$

   and just the disk

$$I_{\text{disk}} = I_{\text{cross+disk}} - I_{\text{cross}}$$

   .

9. Calculate the moment of inertia of the disk by measuring its mass, M and radius, r and computing

$$I_{\text{disk,theoretical}} = \frac{1}{2}Mr^2.$$

10. Do the same for the ring

$$I_{\text{ring,theoretical}} = Mr^2.$$

11. Compare the two sets of moments of inertia

Chapter 9

Simple Harmonic Motion

9.1 Introduction

A simple pendulum consists of a mass which hangs via a string from a fixed point at which the string is attached. Ideally, the string has no mass and the object with mass has no size (*i.e.* a particle). We can then fully specify the pendulum with very few pieces of information. All we need is the mass of the object M, the length of the pendulum L, and the angle from which we release it θ. These data specify the pendulum, but in this lab we'll be measuring the time it takes from release of the pendulum until it repeats the same motion, namely the period T.

We study pendulums in part because of this simplicity but also because of the ubiquity of their motion. In particular, the pendulums exhibit simple harmonic motion. Such motion can be characterized by the functions sin and cos. For example, if you were to shine a flashlight directly down on the pendulum bob, the position of the shadow, x, would vary in time as

$$x = A \sin \omega t.$$

So, x would "swing" from A to $-A$ and back again, repeating the motion in time. The amplitude of the swing is A. The parameter ω is called the *angular frequency* because when you multiply by t you get an angle (with units of radians). To see that things repeat, take the sine of $\omega t = 0$ and sine of $\omega t = 2\pi$.

The parameter more common to everyday language is (just plain old) *frequency*, f. Frequency f is just the rate of repetitions for which the usual unit is cycle/s. However, cycle isn't a *real* unit so this is really just $1/s$. To avoid some confusion (to some people, writing $1/s$ looks strange), we have a name for this unit called the Hertz, Hz $= 1/s$. Angular frequency is related to regular frequency by $\omega = 2\pi f$ and has units of radians/s. Again, radians aren't a real unit either. The final term is the *period*, T which is the time it takes for a cycle to repeat. You should be able to figure out that it is related to frequency by $T = 1/f$.

Understanding simple harmonic motion and the associated variables (T, f, ω, etc.) is not just to help you throughout your lives with pendulums! Such motion is found throughout much of science, and indeed through much of your everyday lives. You hear resonances (from the body of a violin to the harsh sound of singing in the shower) all the time. Engineers have to "tune" resonances when building just about anything from a bridge to your car so that things don't break.

Some experimental issues to which you should pay attention:

1. When measuring the length of the pendulum, you have to consider from what and to what you should measure the length. From the start of the object? The end of the object? The middle of the object? As a hint, try to answer the question: *what length for an ideal pendulum would yield the same period as that you measure for your non-ideal pendulum?*

2. When mounting the string make sure that the point at which you attach it remains fixed. It often happens that your first attempt will allow the string to spin around the mount. This will throw your results off.

3. When measuring the period, you can probably think of lots of sources of error. However, if you measure the time for multiple periods and divide by that number to get actual period (say 10), you might be able to minimize that error. However, what's the best number of repetitions to time? If you time a large number (say 100), you might run into other problems. As is usual, you need to find a good balance.

4. Keep in mind that you will be altering various parameters of the pendulum as you go, so it pays to plan a bit how you will adjust things for each of the following sections.

5. Because we're looking at how the period is affected by physical changes to the pendulum, it makes sense to get a good grasp of the randomness of the periods we measure. In other words, say you measure the

period of a pendulum to be $1.323s$. Now, don't change anything, but simply repeat the measurement. What are the chances of measuring precisely the same value again? By repeating the same measurement a few times, you should have a good idea what your true uncertainty in measurements of T are.

9.2 Procedure

9.2.1 Dependence of Period on Θ

1. Pick and record a length for the pendulum.

2. Pick and record a mass for the pendulum.

3. Pick a starting value of θ, say 10^o and pull the string back to that angle.

4. Release the object and determine the period.

5. Choose a large value of θ, and repeat. You should be able to choose a range of values, perhaps $\theta = 5°, 10°, 15°, 20°, 25°$, and then $50°$. Determine the period for at least five different values.

6. Try to quantify the dependence of the period of a pendulum on the angle from which it is released. In other words, try and reduce your data to a single number (preferably a percentage) which represents how much the period changed when you changed θ.

7. You should also keep everything the same (L, M, and some value of θ) and measure the period a few times. This should give you an idea of how much you can expect the period to change just from the randomness of your method.

9.2.2 Dependence of Period on M

1. Pick and record a length for the pendulum.

2. Pick and record a θ for the pendulum.

3. Pick a starting value of M.

4. Release the object and determine the period.

5. Choose larger values of M and repeat (for at least 5 different masses). Remember not to change the length while you change the mass.

6. Try to quantify the dependence on the mass of the pendulum bob.

9.2.3 Dependence of Period on L

1. Pick and record a M for the pendulum.

2. Pick and record a θ for the pendulum.

3. Pick a starting value of L.

4. Release the object and determine the period.

5. Choose larger values of L and repeat (say for values 5, 10, 15, 20, 25, 30, 35, 40, 45, 50 cm). Choose lengths that cover as wide a span as is feasible...until the pendulum is hitting the floor!

ANALYSIS:

1. For small displacements, theory predicts that the period is related to the length of the pendulum by

$$T = 2\pi\sqrt{\frac{L}{g}}.$$

2. Plot some function of T versus some function of L for this data so that the theory predicts a line.

3. Add a best fit line to the graph and compute the slope of this best fit line with appropriate units.

4. From the value of this slope, compute an experimental value of g and determine its uncertainty from the error in the fit.

Chapter 10

Waves on a String

10.1 Introduction

This week we encounter yet another ubiquitous physical phenomenon, *waves*. It goes without saying (but I state in anyway) that waves are everywhere we look. Two of our senses depend on waves to inform us of our world, light and sound waves. The dynamics of the ocean are dictated in large part by waves. We can consider the electrical pulses that drive our heart as the sum of various waves.

With this brief justification of our study of waves, let me also add that waves are yet another example of harmonic motion.

10.2 Properties of Waves

Imagine a taught string stretched between two points (perhaps a violin or guitar string). If we prick the string then a disturbance travels in both directions up and down the string.

What happens when that disturbance hits the ends of the string where the string is tied down? You should be able to realize that the end points themselves can't move. It turns out that the disturbance "bounces" (*reflects*) off these end points much as a rubber ball bounces off a wall. Unlike the ball however, the disturbance "flips" over when it bounces.

If we then continue to prick the string, then the "positive" disturbances are traveling in one direction while the "negative" disturbances (the flipped over, reflected disturbances) are traveling in the other direction. These dis-

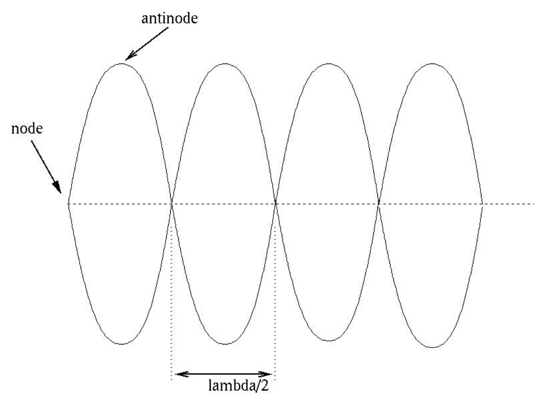

Figure 10.1: Schematic of a standing wave pattern.

turbances add together (superposition) as they pass each other and so, in many places, they cancel out (where they have opposite sign) and in other places add together. These two possibilities represent *interference*, either *constructive* or *destructive*.

The destructive interference is the idea behind machines made to create silence. For example, you can buy a device with headphones which uses a microphone to detect sound waves that are about to hit your ears. The machine then electronically flips the sound, reproducing the negative of the incoming sound through the headphones. The ambient sound interferes with the sound produced by the headphones so that the two waves cancel out (interfere destructively). If the device could work perfectly, then you would hear nothing!

Under special circumstances, disturbances and their reflections propagate to form what is called a *standing wave* (see Fig. 10.1). Here, a pattern is formed because at any point on the string, the disturbance and the reflection always add together in the same way. For such a pattern, there are points which do not move (experience complete destructive interference), called *nodes* and points which move more than any other points (complete constructive interference), called *antinodes*.

Standing waves, when they occur, are an example of resonance. Just as a pendulum has a natural period, as does the spring, a string will resonate (*i.e.* form standing waves) only for certain frequencies. Imagine singing in a bathroom. It doesn't generally sound good because of the reflections. Indeed if you sing at certain frequencies, the sound might be particularly intense if that frequency forms a standing wave. Why is the bathroom more likely to produce resonance than, say, your living room?

We have already discussed frequency, angular frequency, and period in previous labs. Here, we add wavelength, λ. For a standing wave, the wavelength is just twice the distance between two nodes (or antinodes).

Fig. 10.1 helps clarify the conditions under which the string will demonstrate standing waves. The condition is that half wavelengths "fit" on the string between the two end points. So if the length of the string is L and a half-wavelength is $\lambda/2$, then the condition for standing waves is that

$$L = n\left(\frac{\lambda}{2}\right) \qquad n = 1, 2, 3, \ldots$$

where n is just an integer representing the number of half-wavelengths. We also have the relationship between wave speed v and frequency as

$$\lambda = \frac{v}{f}.$$

The speed of the wave v is dictated by the tension in the string T and the string's linear mass density μ. For $n = 1$, the string oscillates at its lowest natural frequency which is called the *fundamental frequency* of the string or, equivalently, its *first harmonic*. For larger values of n, the string oscillates at higher harmonics (*e.g.* the second harmonic).

As an example of harmonics, consider a trumpet, a piano, and a computer playing a specific note, say middle C. These are all the same note, but they certainly don't sound the same. Musical instruments are tuned to resonate at the frequencies of various notes. In a piano, the strings have just the right length and tension for the desired note whereas in a trumpet you press valves which change the length of the tube so that a note is in resonance. These instruments also generate higher harmonics, but at much less loudness than the fundamental. What differentiates the various instruments is the relative loudness of these higher harmonics. The computer generated tone may not generate any higher harmonics, which would sound very tinny and harsh.

10.3 Procedure

It turns out for a string that the various parameters are related by

$$\lambda = \frac{1}{f}\sqrt{\frac{T}{\mu}}.$$

1. If you are **not** given a density for the string, then: COMPUTE μ FOR THE STRING: Cut a piece of string about $1.5m$ long. Record its mass m and length L_0 and then compute the linear mass density $\mu = m/L_0$. From σ_m and σ_{L_0}, calculate σ_μ. Also record what kind of string you're using (kite string, black nylon string, etc).

2. SETUP THE VIBRATING STRING: Connect the string to the string oscillator and to a pulley clamped to the other end of the table. Ensure the string is level, and that the oscillator and pulley are well-aligned with each other.

3. ADD THE MASS: Allow the string to hang over the pulley and attach a mass hanger.

4. GET FAMILIAR: By changing the amount of hanging mass, try to get the string to resonate. Experiment with whether adding mass increases the number of nodes or decreases the number.

5. FINAL SETUP: Get the string to have 7-9 loops. When you have a standing wave pattern, record the following:

   (a) n – the number of loops in the patterns

   (b) m_s – the suspended mass, and its uncertainty

   (c) l_n – the length of one loop in the pattern, and its uncertainty

   Repeat this for decreasing numbers of loops.

6. MEASURING THE LOOP LENGTH, l_n: Measure l_n as the distance from where the string contacts the pulley (a node) to the node closest to this point. Measure its uncertainty.

7. The frequency of the oscillator, f, should be twice the frequency of the wall current, 120 Hz.

8. COMPUTE: compute the following:
   (a) T – the tension in the string, $m_s g$, and its uncertainty
   (b) λ – the wavelength of the standing wave, $2l_n$, and its uncertainty

9. ANALYSIS: Again, you'll need a computer here. From Eq. (10.3), find appropriate quantities (*e.g.* $\sqrt{\lambda}$, λ^2, $1/T$, etc.) to plot such that you expect a linear relationship. On such a plot, include error bars on the relevant quantities, add a best fit line. From the slope of the best fit line extract an experimental value of the string density μ. Compare μ to that already measured (or given).

10. DISCUSSION:
    (a) Did the string demonstrate standing wave patterns?
    (b) Did your data follow the behavior predicted by Eq. (10.3)?
    (c) Which data points are more trustworthy: the large l_n values or the small? Why?
    (d) What's the biggest source of error? How could you reduce it?

Chapter 11

Speed of Sound in Air

11.1 Introduction

We have studied vibrating pendulums, springs, and strings, and seen what natural frequency and resonance mean. Today, our quest is to measure the speed of sound, and to do so, we utilize the idea of resonance once again. However, here it is the air itself that will be vibrating.

The speed of sound depends on temperature, but at 20° Celsius it is approximately 344 m/s. That's pretty fast and you might imagine it would be difficult to measure directly. Indeed, we use the wave properties of sound to determine its speed.

Sound propagates as a pressure disturbance in air that our ears detect. Sound also obeys the relationship we saw earlier with the string

$$v = \lambda f \tag{11.1}$$

where λ is once again the wavelength and f is frequency. This formula should be easy to understand. Picture one cycle (*i.e.* one wavelength) of a wave that moves to the right. This section takes a time $T = 1/f$ to travel the distance λ. So, using the simple formula $v =$ distance/time, you get $v = \lambda/T$ which gives the relationship (11.1) above.

So, if we can find the wavelength and frequency for a sound wave, we can just multiply them to get the speed. Determining frequency is relatively easy: we just buy a tuning fork with the frequency stamped right on it! We can get the wavelength in a manner analogous to what we did with the string. We simply create resonance and use the properties of standing waves to determine the wavelength.

11.2 Procedure

To create resonance, we need an enclosure whose length we can vary (*i.e.* tune). We use a long, vertical glass tube connected to an adjustable source of fluid (here, water). By adjusting the height of the water in the column, we change the length of the air column. By placing a tuning fork near the top, we can send sound waves down the tube which will reflect upwards. When the air column has the right length, we should get resonance. From the length of the tube, we can determine the wavelength.

We need to determine how the wavelength is related to the air column length L. For this you have to picture the standing wave pattern. At the bottom of the air column where the water stops the air has no place to go and there is thus a node there. At the top of the column you would expect an antinode because the air is free to oscillate there. This would imply that the integer numbers of quarter wavelengths would need to fit into the tube to create a standing wave pattern because the distance between a node and an adjacent antinode is $\lambda/4$.

However, because of effects due to the diameter of the tube, the antinode will generally lie just a bit above the tube. So instead of taking the length of the column tube and multiplying by 4 to get the wavelength, we'll do something else to get a better value for the wavelength. We'll find the distance between the antinode and where the next antinode appears. This distance should be $\lambda/2$.

1. Pick a tuning fork and record the frequency f stamped on it. Assume that σ_f is negligible.

2. Strike the fork with a mallet to get it oscillating. Don't hit the fork with anything hard or you will damage it. Also, don't hit the fork too hard so that you don't excite a higher harmonic than the frequency specified on the fork.

3. Place the fork vertically just above the opening of the tube.

4. Adjust the height of the water column by lowering the input tube. Carefully listen for resonance as you lower the water level. Determine the length L_1 of the water column when you hear the first resonance.

5. Continue to lower the water level until you hear a second resonance point. Record and determine the length of the column for the second resonance L_2. Note the uncertainty in determining L_1 and L_2.

6. Compute the wavelength via $\lambda = 2\,(L_2 - L_1)$. Compute σ_λ.

7. Compute the velocity of sound in air via $v = \lambda f$. Compute σ_v.

8. Repeat for two other tuning forks with different frequencies.

9. Measure and record the temperature of the room, T, in Celsius, and its uncertainty.

10. Using the formula:

$$v = 331.5\frac{m}{s} + 0.607\frac{m}{s\,^{\circ}C}T$$

compute the accepted value of the speed of sound v. Compute the uncertainty in this quantity.

Appendix A

Error, Uncertainty, and Significant Digits

A.1 Concepts Introduced

uncertainty; error; systematic error; random error; significant digits; percent error; precision; accuracy; uncertainty propagation; mean; standard deviation; chi square

A.2 Errors

By the term **error**, I mean the difference between a given measurement or assertion and the accepted value. I am 5' $9\frac{1}{2}$" tall. If you tell me that I am 5' 8" give or take a half inch, then the error is $1\frac{1}{2}$". Thus, I differentiate between the terms error and uncertainty. Other scientists and books do not, so you'll have to look at context. In particular, you should be aware that the study and use of uncertainties in the lab is commonly called *Error Analysis* because uncertainty and error are intimately related.

Random errors do not mean simply unexplained workings of the universe. Instead, it refers to errors with definite causes but whose effects occur with arbitrary sign. For example, wind from an open window might affect a measurement on a mass balance. If the wind comes at random times with random pressures (i.e. it could be blowing on one side of the scale at one time and the other at another time), then this would be random error. Taking many measurements and averaging is often good at reducing the amount of random error (why?), but this is not always possible.

Systematic errors are the errors present which have effects with the same sign. For example, measuring the acceleration of a cart down an inclined air track, air drag would, in general, represent a systematic error tending to make the observed acceleration *less* than would otherwise be expected if there were no friction.

Personal errors are those committed by you. For example, if you write in your lab "We forgot to level the platform and this caused error," then that is a personal error. It happens to be one for which you have no excuse; you should have followed instructions and seen the error before you left lab so that you could fix it.

A.3 Uncertainties

Any measurement is going to have inherent uncertainty. If you get on a digital scale, and it says you weigh 150 lbs, it is obviously not exact. Some other scales will be more precise and others won't. However, they might all say 150 lbs. How do we indicate what precision we have with a given instrument?

First, I want to differentiate between **precise** and **accurate**. Let's say we have two scales which are completely accurate. In other words, we can trust completely what they say. However, scale (A) is more precise than scale (B). Let's also assume that you weigh exactly 150 lbs. Then scale (A) has a lower uncertainty than scale (B) and might read something like 150.000000 ± 0.000001 lbs. Scale (B), being less precise, would then have a much higher uncertainty perhaps comparably to the typical bathroom scale 150 ± 1 lbs. Precision, as I use it, has nothing do with accuracy.

Now consider a similar situation in which two scales have the same precision. Scale (C) reads 150 ± 1 lbs and scale (D) reads 155 ± 1 lbs. How can that be? We're still under the assumption that the true weight is 150 lbs. Well, we describe this situation by saying that scale (D) is less accurate than scale (C). In fact, scale (D) is simply inaccurate. However, their precision is still the same.

To get a better idea why every measurement has uncertainty, just imagine the most precise measurement you can. Write it down. For example, if I measure the length of something to be $1.2345678901234567890 \pm 0.00000000000000000005$ cm, that's quite precise. The uncertainty is very,

very small. But yet, this measurement can't differentiate 1.23456789012345678903 cm from 1.23456789012345678997 cm.

To define terms, consider a measurement reported as $E = 1.234 \pm 0.001J$. The **absolute uncertainty** in the measurement is denoted $\triangle E$ and here takes the value $\triangle E = 0.001J$ (notice that the absolute uncertainty takes the same units as the measurement). The **fractional uncertainty** is the ratio of the absolute uncertainty to the value itself and is denoted by $\frac{\triangle E}{E}$. Here it takes the value $\frac{\triangle E}{E} = 0.0008$ (or 0.08%; notice it has no units).

Unless otherwise noted, I want only one significant digit (and not more than one) to denote either type of uncertainty. The uncertainty itself is only approximate and thus it is confusing to put more than one digit for the uncertainty.

A.4 Significant Digits

You should have experience with the concept of significant digits already, but here is a brief review. In lab, unlike perhaps in other places, when you present a number, such as by writing it in a lab write-up, you are asserting that you know the number to be exactly as you write it. So if you use a ruler to measure a length, let's say, of $3.45 inches$, and write $3.4500 inches$ you are, in the very least, being very misleading. At the worst, you are being outright dishonest.

So let's say you're adding two measurements, $3.45 inches$ and $0.123 inches$, to get, for example, a total length of something. How do you do it? Well, you add the two as you normally would, and then consider the rules of significant digits. The rule for adding numbers is that you start from the left keeping all digits until rounding when you reach the place where a digit has no more significant digits. So you'd add normally to get 3.573, and then, starting from the left, you'd keep $3.57 inches$ because the first number has no more significant digits after the hundredths place. Since the next digit is 3, you don't need to round up. If you add $3.45 inches$ to $0.127 inches$ you would instead get $3.48 inches$. The reason the rule works like this is because if you don't know the thousandths place digit of the first number, then it doesn't make sense to report a digit of 3 there in your summation. You simply don't know what goes there, so reporting something would be a mistake. These same rules apply for subtraction.

Here's a tougher example. Consider the summation below where I denote digits we simply don't know by an X (as opposed to leaving it blank like we usually do):

$$\begin{array}{l} 1.0XXXXXX \\ 0.1XXXXXX \\ 0.11XXXXX \\ 0.111XXXX \\ 0.1111XXX \\ 0.11111XX \\ 0.111111X \\ ------------ \\ 1.654321X \end{array}$$

But what is 1 plus X? We don't know. So the answer can only extend to the tenths digit because we don't have any Xs in that place. But instead of it being simply 1.6, we have to round to get 1.7.

When multiplying (or dividing) two numbers, the rule is different. Consider multiplying 0.10 with 1.23456789. The normal way to multiply gives a result of 0.123456789 but this has too many digits. How many do you get rid of? The rule is simple, though possibly not very intuitive. The number of digits of the result of multiplying is equal to the *least* number of significant digits of any of the numbers multiplied. We're multiplying two numbers, the first has 2 sig-digits, the second has 9 sig-digits. So the lesser of these is 2, and so the final result has only 2 significant digits. The result is then 0.12. Notice that which place the digits are in (tenths, hundredths, etc) doesn't matter.

One last thing. When computing something complicated such as $(0.1 + 0.05) \times 8$ be careful to keep all digits possible so that you don't introduce rounding errors. For example, you might think the answer to the following would be obtained by doing the following

$$\begin{array}{r} (0.1 + 0.05) \times 8 \\ 0.2 \times 8 \\ 2. \end{array}$$

Here all the rules above were followed, but we got the wrong answer. Instead we do the entire computation with all digits possible, but we'll keep track of the sig-digits separately in brackets:

$$(0.1 + 0.05) \times 8$$

$$\begin{array}{r} 0.15[0.2] \times 8 \\ 1.2[1]. \end{array}$$

So the right answer is 1. In other words, always wait till the end of a computation to round.

A.5 More Worked Examples

1. What is $(1.30 \times 10^3) + (0.7 \times 10^1)$?
   This is easier if we right it out as

   $$\begin{array}{r} 1300 \\ +7 \\ ---------- \\ 1307 \end{array}$$

   Here we've added normally, but now we have to figure out how many digits to report. So let's look at each of the digits of our preliminary answer 1307. Starting from the left, the 1 is okay because we know all the digits before the 7 are just zero. The same reasoning says that the 3 is fine. What about the zero? Well, we know zero plus zero is zero, so we're fine. What about the 7? Turns out that we can't report this, but why? We can't report it as a seven because we don't know that $0 + 7$ is zero because we don't know that the zero we wrote in the 1300 line is a zero (when we converted from scientific notation, we threw the zero in to hold the place, but it's actually an X!).
   So we should have worked the problem like this:

   $$\begin{array}{r} 130X \\ +7 \\ ---------- \\ 130X \end{array}$$

   and it would be clear the answer is 1.31×10^3.

2. What is $5.4 \times 12,345$?
   Plug into calculator to get: $66,663.00$. But that's too many digits. So look at the two numbers being multiplied; they have 2 and 5 significant digits. Because 2 is smaller than 5, we know we should only report two digits in our answer. So we round and get 6.7×10^4.

3. What is $0.344 \times (11.0 + 0.2345)$?

$$
\begin{aligned}
0.344 \times (11.0 + 0.2345) \\
0.344 \times (11.2) \\
3.86
\end{aligned}
$$

   If you got 3.85 that's not right. Keep all digits for the final computation: $0.344 \times 11.2345 = 3.8647$ and then round (**not** $0.344 \times 11.2 = 3.8528$ and round).

4. What is:

$$\frac{(0.345 + 12) \times (0.001 + 0.005)}{(1.3 + 1.2) \times 4}?$$

   Work out on calculator first to get: 0.007407000. Now go back to figure out right number of digits:

$$
\begin{gathered}
\frac{12 \times 0.006}{2.5 \times 4} \\
\frac{0.07}{1 \times 10^1} \\
0.007
\end{gathered}
$$

   This tells you that you only have one significant digit. So now round the exact answer to one digit and you get 0.007.

A.6 Questions to be answered

For the following, report with the correct number of digits and with the appropriate uncertainties.

1. Using a balance in the lab, I measure the mass of a comb to be $9.3 \pm 0.4g$.

   (a) What is the *absolute* uncertainty?

   (b) What is the *fractional* uncertainty?

2. Which of the following makes the most sense:

   (a) $1.1kg \pm 0.1N$

(b) $1.1 \pm 0.11231 N$

(c) $1.1 \pm 10.1 kg$

(d) $1.1 \pm 0.1 kg$

3. How many significant digits are in the measurements: 1.234 inches, 1.5×10^{-3} miles, 1300.1 seconds?

4. With the proper number of significant digits, what is the product of 0.012 and 1.234×10^5?

5. What's the result of

$$\frac{0.41 \times 0.00111}{(5.3 + 0.65) \times (0.0011 - 0.0002)}$$

6. What's the fractional uncertainty of the measurement 5.293 ± 0.001 lbs? Of 10.30 ± 0.001 lbs? Which has the higher absolute uncertainty? Which has the higher fractional uncertainty?

A.7 Acknowledgments and References

Some of this material follows closely that of *Measurement and Errors*, prepared by Radh Achuthan. The book *An Introduction to Error Analysis* by John R. Taylor also serves as a good reference.

Appendix B

Notes about Writeups & Graphs

B.1 What I Want

Students always ask, "What do you want in the writeup?" If it were up to me (wait, I guess it is up to me!), I would prefer to leave that completely up so long as you convey in it the data, what you did with the data, and what you can conclude from the data. It's really that simple, but people always want more constraints. So here goes....

- **point** – In one sentence, describe what the purpose of the experiment is. Try to avoid bias (*e.g. we are proving that energy is conserved*). Good, non-biased action verbs are *test*, *investigate*, *measure*, and *determine*.

- **data** – Present your data. Except in unusual circumstances, you need to provide original data. For example, if you measure a bunch of times for a cart to cross some distance and then compute a velocity from that, you must include those original times, not just the computed quantities. You must try to present your data in a logically organized table(s).

- **sample calculations** – I do **not** want to see every computation you do. Only a single instance of each type. I only look at this if I suspect you computed something incorrectly.

- **discussion/conclusion** – This is the hard part. It is this section that will likely need the most work. The good news is that I hope that everyone's discussion improves throughout the year. At the very least, my standards as far as what I expect will increase. First, let me note that there is only one section here...you do not need to reiterate what you say in a separate conclusion section. When you write this section, imagine that someone that took the class last year is reading it and that they are reading it first before having looked at the data. They know about physics, but they're not sure you expected, nor what you saw. You should mention a few of the results from the data/calculations section along with what they mean. For example, you might say:

  > The average difference between the final and initial velocities was 8.3%. If there were no friction or air drag, we would have expected this difference to be non-existent, and therefore it seems reasonable to expect the effects of friction and drag to lead to an 8% velocity drop.

  This is much better than

  > The initial velocity in the first run was $8.2 mm/s$ and in the second run was $6.5 mm/s$. The final velocities were $7.6 mm/s$ and $5.6 mm/s$. These are obviously different so the velocity changed. Possible errors include the friction, drag, heat, humidity, and personal error.

  Do not (1) overload the reader with data values, (2) give a laundry list of possible errors with no real explanation of how they would affect that data values, (3) mention personal error.

In case it is not clear from the above, you do not need, and I will not read any discussion of materials or procedure. We all know what we did and what equipment we used. Also, these lab writeups should only be about 2 pages and should be mostly written by the time you leave the classroom. In fact, you should only need to spend at most an hour outside of class to complete the writeup.

B.2 Random issues

Reiterate important numbers in the paragraphs of your analysis so that the reader doesn't have to go back to the data to find the numbers. Be careful not to put too many numbers in the text or it gets confusing.

Any and all measurements you take need to be in the writeup. So if you measure something in one unit, and then convert to another unit, make sure you record the original data. The idea here is that someone should be able to reproduce all the steps you take after you measure the data if given just your writeup.

Be careful when using "small", "bigger", and other words that are meant to compare things. Be explicit in terms of what you're comparing against. For example, if I were to write:

> Our error was 10% which is very small. Because it is small, our test helps confirm Liebling's law.

How is the reader to know that 10% is small? Small compared to what? It would be better to write:

> Our error was 10%. As discussed above, the wind provided an unexpected source of error for which we could not account in our analysis. However, it seems reasonable to expect that it likely could account for such an error. In that case, these results support the validity of Liebling's Law.

Include all original data. If you compute something from some measurements, do not just show me the computed quantity. The original data is perhaps the most important part of the lab.

Write for an audience that might read a journal. Write not that you were told to do something or that you did this to learn (both of which are true). Write as if someone *should* be interested in reading it.

Discussing error:

1. What is a source of error?

2. Will it actually affect your results? For example,

3. How big is it?

4. Does the expected effect actually correlate with your observed error?

5. Can you estimate the size of the error? If so, compare the estimate to your observed error.

There is always the possibility that you made a mistake, what people appear to call "human error." Despite this possibility, do not mention it in the writeup. If you suspect you made mistakes, it's your obligation to track them down not excuse them in the writeup. If everything appears correct, then do the writeup and I'll look for mistakes. In the real world, your paper would get peer-reviewed and those reviewers would point out mistakes. But you don't see in journals, "well, maybe we made mistakes measuring, calculating, etc."

B.3 Guide to Writeup Grades

As somewhat of a guide for what is at least somewhat subjective, some instructors might use a scale for lab grades as follows:

- 6 – Was present for lab, but writeup very incomplete
- 7 – Writeup has serious flaws and/or did not complete the purpose
- 8 – Attempted purpose but didn't quite succeed or didn't follow all directions
- 9 – Succeeded at purpose but still has certain minor flaws
- 10 – Succeeds at purpose and succeeds at presentation in a convincing manner

B.4 Making Graphs

A graph that plots "A vs. B" means that B is measured on the horizontal axis while A is measured on the vertical.

Graphs require appropriate units on the two axis.

Data points should be plotted as just that, points. They should be clearly visible. Regardless of what else appears on the plot (*e.g.* a curve which fits the data), the point have to be visible so that the reader can evaluate them (*e.g.* how well the curve fits them).

Make the plot as big as is convenient. If you have plenty of space on the page, make it big. It's hard to tell much from a small graph.

B.5 Extracting Information from a Fit

You will need to be able to study the relationship among variables using a graph and best fit line as tools.