February 3, 2009

General Relativity

Review Special Relativity

Key Ideas of Special Relativity

- No material object can travel faster than light
- If you observe something moving near light speed:
  - Its time slows down
  - Its length contracts in direction of motion
  - Its mass increases
- Whether or not two events are simultaneous depends on your perspective

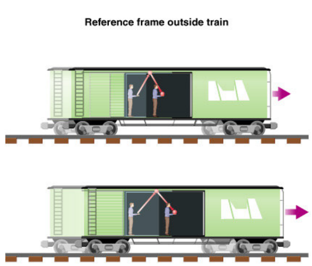
Absolutes of Relativity

1. The laws of nature are the same for everyone

2. The speed of light is the same for everyone

All of relativity follows from these two ideas!

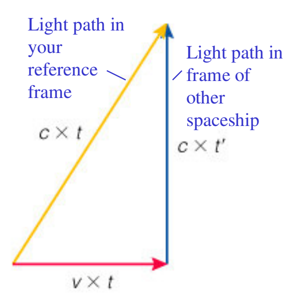
Path of Ball in a Moving Train



- Someone outside the train would see the ball travel a longer path in one up-down cycle

- The faster the train is moving, the longer that path would be

The Time Dilation Formula



$$c^2 t'^2 + v^2 t^2 = c^2 t^2$$

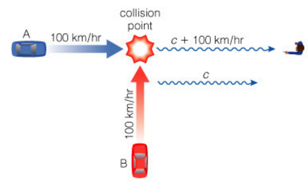
$$t'^2 = t^2 - \frac{v^2}{c^2} t^2$$

$$t' = t\sqrt{1 - \left(\frac{v^2}{c^2}\right)}$$

Tests of Relativity

- First evidence for absoluteness of speed of light came from the *Michaelson-Morley Experiment* performed in 1887
- Time dilation happens routinely to subatomic particles the approach the speed of light in accelerators
- Time dilation has also been verified through precision measurements in airplanes moving at much slower speeds
- Prediction that $E=mc^2$ is verified daily in nuclear reactors and in the core of the Sun

A Paradox of Non-Relativistic Thinking

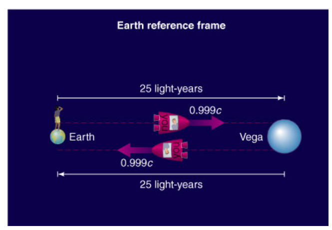


- If speed of light were not absolute, you would see the car coming toward you reach the collision point before the car it struck
- No paradox if light speed is same for everyone

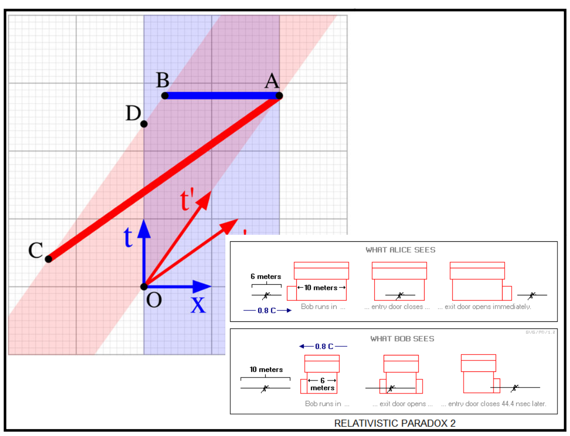
Making Sense of Relativity

- According to you, time slows down in a moving spaceship
- According to someone on that spaceship, your time slows down
- Who is right?
- You both are, because time is not absolute but depends on your perspective

A Journey to Vega



- The distance to Vega is about 25 light-years
- But if you could travel to Vega at 0.999*c*, the round trip would seem to take only two years!



Einstein's Theories of Relativity

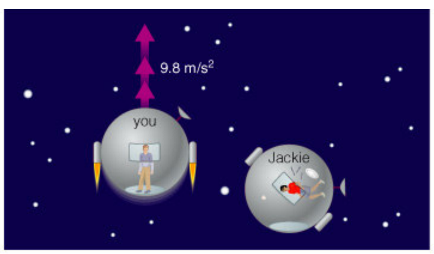
- Special Theory of Relativity (1905)
  - Usual notions of space and time must be revised for speeds approaching light speed (c)
  - $E = mc^2$
- General Theory of Relativity (1915)
  - Expands the ideas of special theory to include a surprising new view of gravity

General Relativity

Spacetime

- Special relativity showed that space and time are not absolute
- Instead they are inextricably linked in a four-dimensional combination called **spacetime**
  - SR joins space with time to form spacetime (but it is flat)
  - GR generalizes SR and allows for curved spacetime

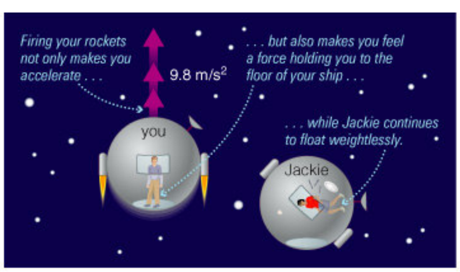
Is all motion relative?



Relativity and Acceleration

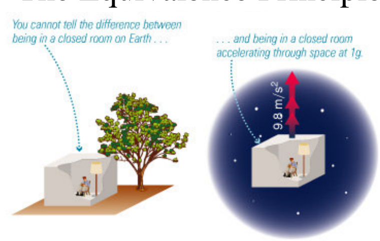
- Our thought experiments about special relativity involved spaceships moving at constant velocity
- Is all motion still relative when acceleration and gravity enter the picture?

Acceleration and Relative Motion



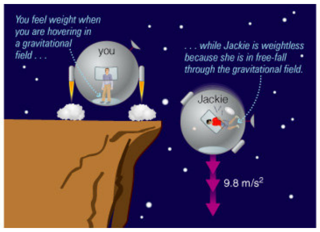
- How can your motion be relative if you're feeling a force causing acceleration?

The Equivalence Principle



- Einstein preserved the idea that all motion is relative by pointing out that the effects of acceleration are exactly equivalent to those of gravity

Gravity and Relative Motion



- Someone who feels a force may be hovering in a gravitational field
- Someone who feels weightless may be in free-fall

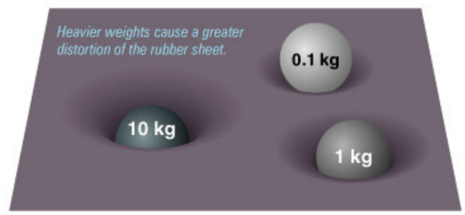
Curved Space

- Travelers going in opposite directions in straight lines will eventually meet
- Because they meet, the travelers know Earth's surface cannot be flat—it must be curved

Curved Spacetime

- Gravity can cause two space probes moving around Earth to meet
- General relativity says this happens because spacetime is curved

Rubber Sheet Analogy

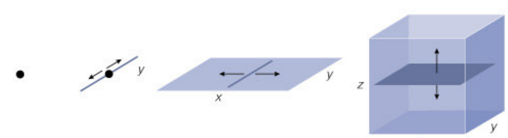


- Matter distorts spacetime in a manner analogous to how heavy weights distort a rubber sheet

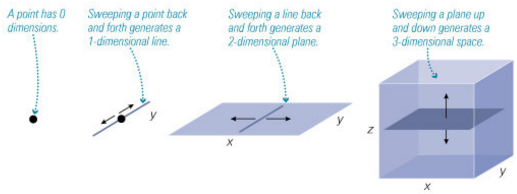
Key Ideas of General Relativity

- Gravity arises from distortions of spacetime
- Time runs slowly in gravitational fields
- *Black holes* can exist in spacetime
- The universe may have no boundaries and no center but may still have finite volume
- Rapid changes in the motion of large masses can cause *gravitational waves*

What is spacetime?



Dimensions of Space



- An object’s number of dimensions is the number of independent directions in which movement is possible within the object

Dimensions of Spacetime

- We can move through three dimensions in space (*x,y,z*)
- Our motion through time is in one direction (*t*)
- Spacetime, the combination of space and time, has four dimensions (*x,y,x,t*)
- Need four numbers to specify out “address” in the Universe….hence 4D

Perspectives in Space

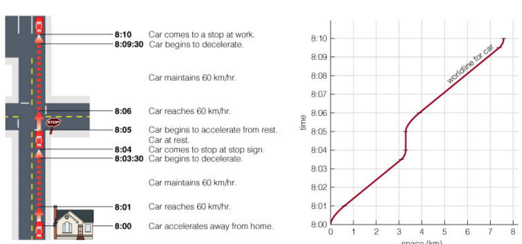
- A book has a definite three-dimensional shape
- But the book looks different in two-dimensional pictures of the book taken from different perspectives
- Similarly, space and time look different from different perspectives in spacetime

Perspectives in Spacetime

- Observers in relative motion do not share the same definitions of *x, y, z,* and *t*, taken individually

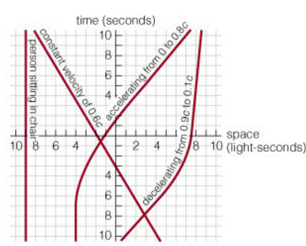
Space is different for different observers.
Time is different for different observers.
Spacetime is the same for everyone.

Spacetime Diagram of a Car



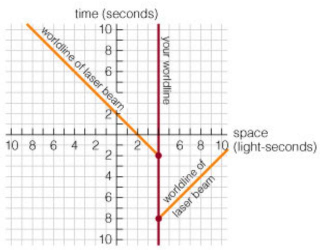
- A spacetime diagram plots an object’s position in space at different moments in time

Worldlines



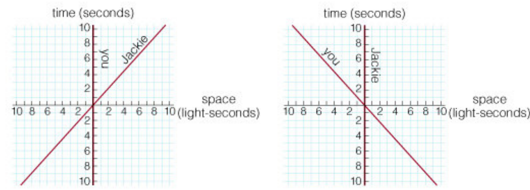
- A worldline shows an object’s path through spacetime in a spacetime diagram
  - Vertical worldline: no motion
  - Diagonal worldline: constant-velocity motion
  - Curved wordline: accelerating motion

Worldlines for Light



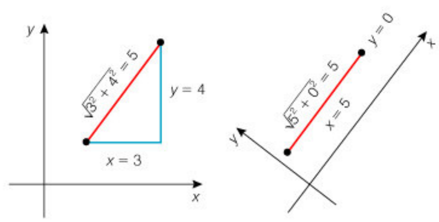
- Worldlines for light go at 45° angles in diagrams with light-seconds on one axis and seconds on the other

Worldlines and Relativity



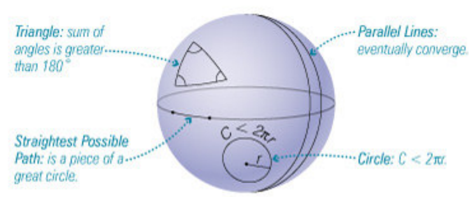
- Worldlines look different in different reference frames

Worldlines and Relativity

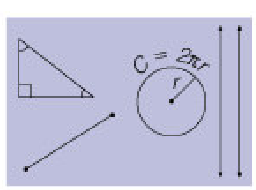


- But everyone will agree on the "distance" between two different events in spacetime: $x^2 + y^2 + z^2 - (ct)^2$

What is curved spacetime?

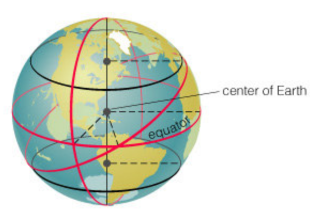


Rules of Geometry in Flat Space



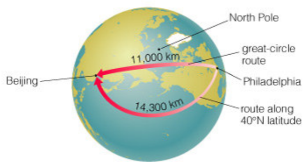
- Straight line is shortest distance between two points
- Parallel lines stay same distance apart
- Angles of a triangle sum to 180°
- Circumference of circle is $2\pi r$

Geometry on a Curved Surface



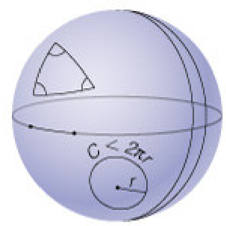
- The straightest lines on a sphere are *great circles* sharing the same center as the sphere
- Great circles intersect, unlike parallel lines in flat space

Geometry on a Curved Surface



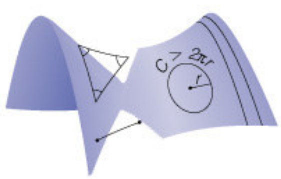
- Straight lines are shortest paths between two points in flat space
- Great circles are the shortest paths between two points on a sphere

Rules of Spherical Geometry



- Great circle is shortest distance between two points
- Parallel lines eventually converge
- Angles of a triangle sum to > 180°
- Circumference of circle is < $2\pi r$

Rules of Saddle-Shaped Geometry



- Piece of hyperbola is shortest distance between two points
- Parallel lines diverge
- Angles of a triangle sum to < 180°
- Circumference of circle is > $2\pi r$

Geometry of the Universe

- Universe may be either flat, spherical, or saddle-shaped depending on how much matter (and energy) it contains
  - Flat and saddle-shaped universe are infinite in extent
  - Spherical universe is finite in extent
  - No center and no edge to the universe is necessary in any of these cases

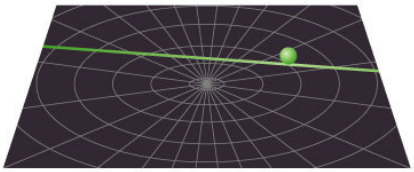
"Straight" lines in Spacetime

- According to Equivalence Principle:
  - If you are floating freely, then your worldline is following the *straightest possible path* through spacetime
  - If you feel weight, then you are not on the straightest possible path

Gravity, Newton, and Einstein

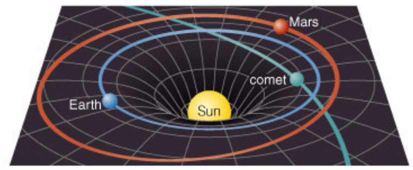
- Newton viewed gravity as a mysterious "action at a distance"
- Einstein removed the mystery by showing that what we perceive as gravity arises from curvature of spacetime

Rubber Sheet Analogy



- On a flat rubber sheet
  - Free-falling objects move in straight lines
  - Circles all have circumference $2\pi r$

Rubber Sheet Analogy

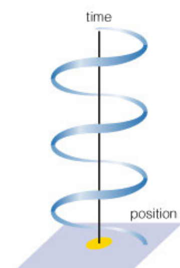


- Mass of Sun curves spacetime
  - Free-falling objects near Sun follow curved paths
  - Circles near Sun have circumference $< 2\pi r$

Limitations of the Analogy

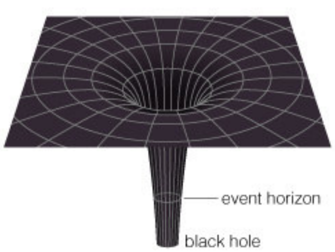
- Masses do not rest “upon” the spacetime like they rest on a rubber sheet
- Rubber sheet shows only two dimensions of space

Limitations of the Analogy

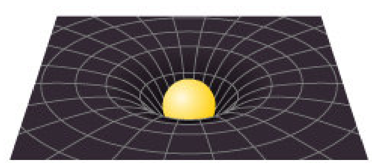


- Rubber sheet shows only two dimensions of space
- Path of an orbiting object actually spirals through spacetime as it moves forward in time

What is a black hole?

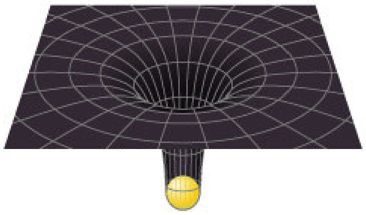


Curvature near Sun



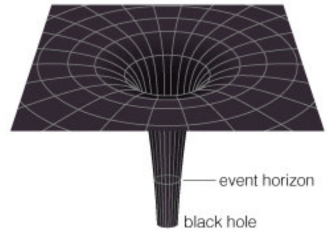
- Sun’s mass curves spacetime near its surface

Curvature near Sun



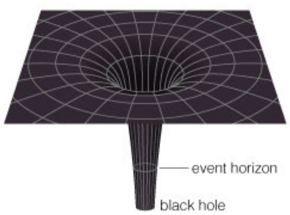
- If we could shrink the Sun without changing its mass, curvature of spacetime would become greater near its surface, as would strength of gravity

Curvature near Black Hole



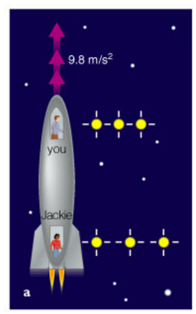
- Continued shrinkage of Sun would eventually make curvature so great that it would be like a bottomless pit in spacetime: a *black hole*

Limitations of the Analogy



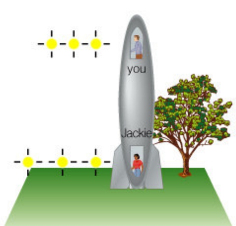
- Spacetime is so curved near a black hole that nothing can escape
- The "point of no return" is called the *event horizon*
- Event horizon is a three-dimensional surface

Time in an Accelerating Spaceship



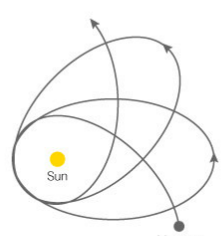
- Light pulse travel more quickly from front to back of an accelerating spaceship than in other direction
- Everyone on ship agrees that time runs faster in front than in back

Time in an Gravitational Field



- Effects of gravity are exactly equivalent to those of acceleration
- Time must run more quickly at higher altitudes in a gravitational field than at lower altitudes

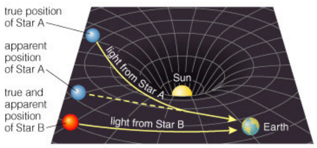
Precession of Mercury



Note: The amount of precession with each orbit is highly exaggerated in this picture.

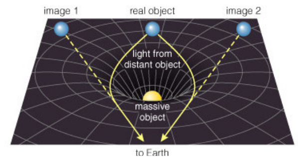
- The major axis of Mercury's elliptical orbit precesses with time at a rate that disagrees with Newton's laws
- General relativity precisely accounts for Mercury's precession

Gravitational Lensing



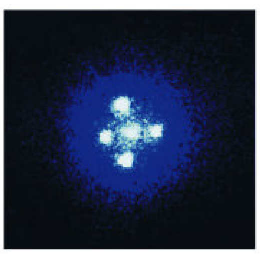
- Curved spacetime alters the paths of light rays, shifting the apparent positions of objects in an effect called *gravitational lensing*
- Observed shifts precisely agree with general relativity

Gravitational Lensing



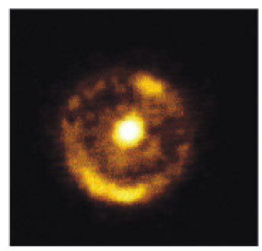
- Gravitational lensing can distort the images of objects
- Lensing can even make one object appear to be at two or more points in the sky

Gravitational Lensing



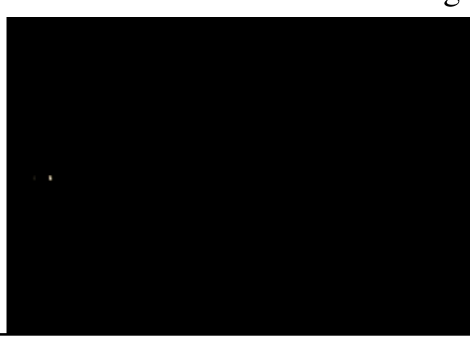
- Gravity of foreground galaxy (center) bends light from an object almost directly behind it
- Four images of that object appear in the sky (Einstein's Cross)

Gravitational Lensing

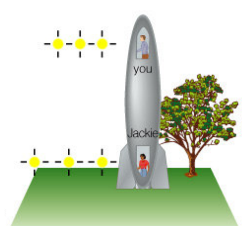


- Gravity of foreground galaxy (center) bends light from an object directly behind it
- A ring of light from the background object appears in the sky (Einstein Ring)

Demonstration of Grav. Lensing



Gravitational Time Dilation

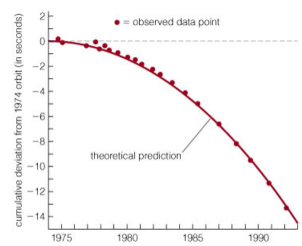


- Passage of time has been measured at different altitudes has been precisely measured
- Time indeed passes more slowly at lower altitudes in precise agreement with general relativity

Gravitational Waves

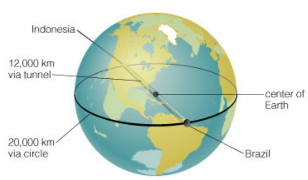
- General relativity predicts that movements of a massive object can produce gravitational waves just as movements of a charged particle produce light waves
- Gravitational waves have not yet been directly detected

Indirect Detection of Waves



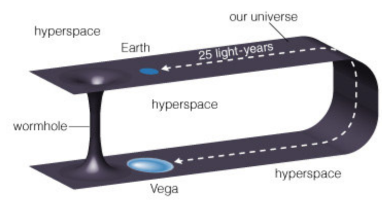
- Observed changes in orbit of a binary system consisting of two neutron stars agree precisely with predictions of general relativity
- Orbital energy is being carried away by gravitational waves

Shortcuts through Space



- If we could somehow build a tunnel through the center of Earth, the trip from Indonesia to Brazil would be much shorter
- Could there be analogous tunnels through spacetime?

Shortcut through Spacetime



- Some mathematical solutions of the equations of general relativity allow for shortcuts called *wormholes* that are tunnels through *hyperspace*

Are Wormholes Really Possible?

- Wormholes are not explicitly prohibited by known laws of physics but there is no known way to make one
- If wormholes exist, then they can be used for time travel
- Time travel leads to paradoxes that some scientists believe should rule out the possibility of wormholes

GR Summary

- Fundamentally based on Equivalence Principle
- Equivalence of acceleration and gravity
- Leads to equivalence of spacetime curvature and gravity
- Elegant but very mathematical equations
- GR Effects (contrasts w/ Newtonian Gravity):
  - Gravitational time dilation (not motion based as in SR)
  - Gravitational Lensing (astronomical tool)
  - Black Holes
  - Gravitational Waves (astronomical tool)
  - Cosmology (we'll cover when we get there)